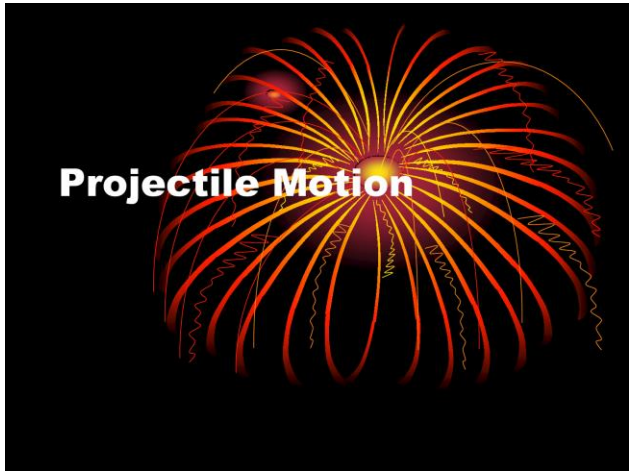
4.3 - Projectile Motion: Launched at an Angle



Projectile Motion

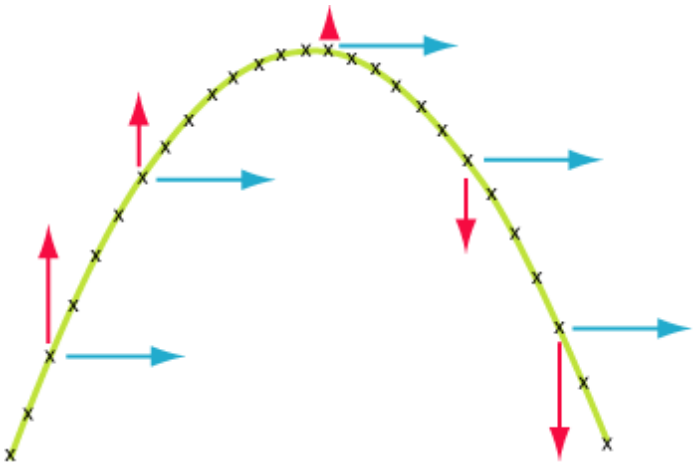
There are two general types of projectile motion situations.

1. **Object launched horizontally**
2. **Object launched at an angle**

Projectiles move in TWO dimensions

Since a projectile moves in 2-dimensions, it therefore has 2 components just like a resultant vector:

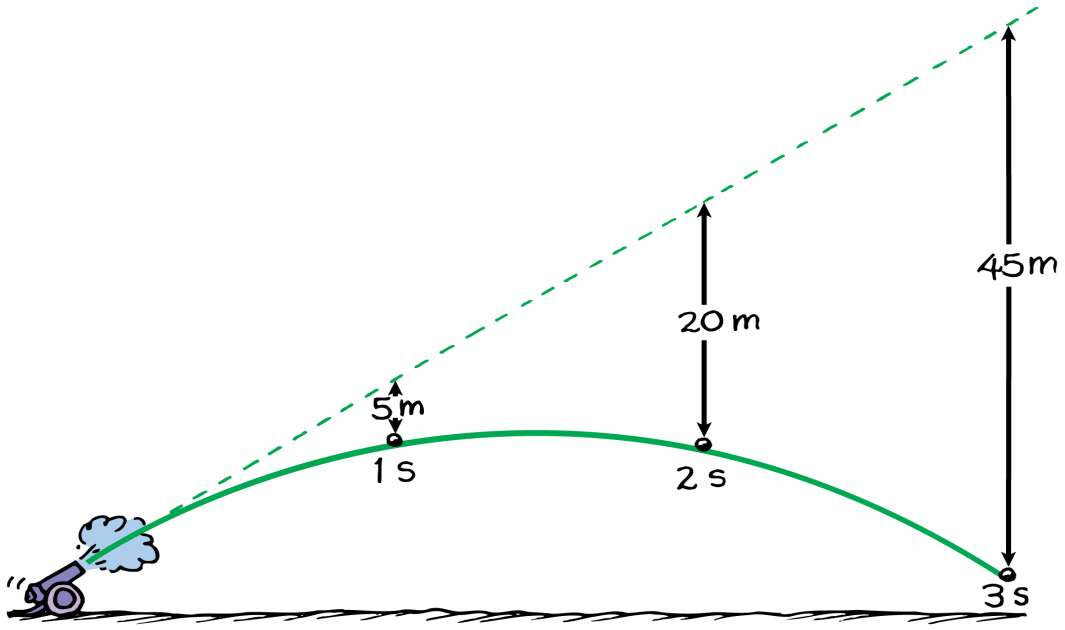
- **Horizontal**
- **Vertical**



Projectiles Launched at an Angle

With no gravity the projectile would follow the straight-line path (dashed line).

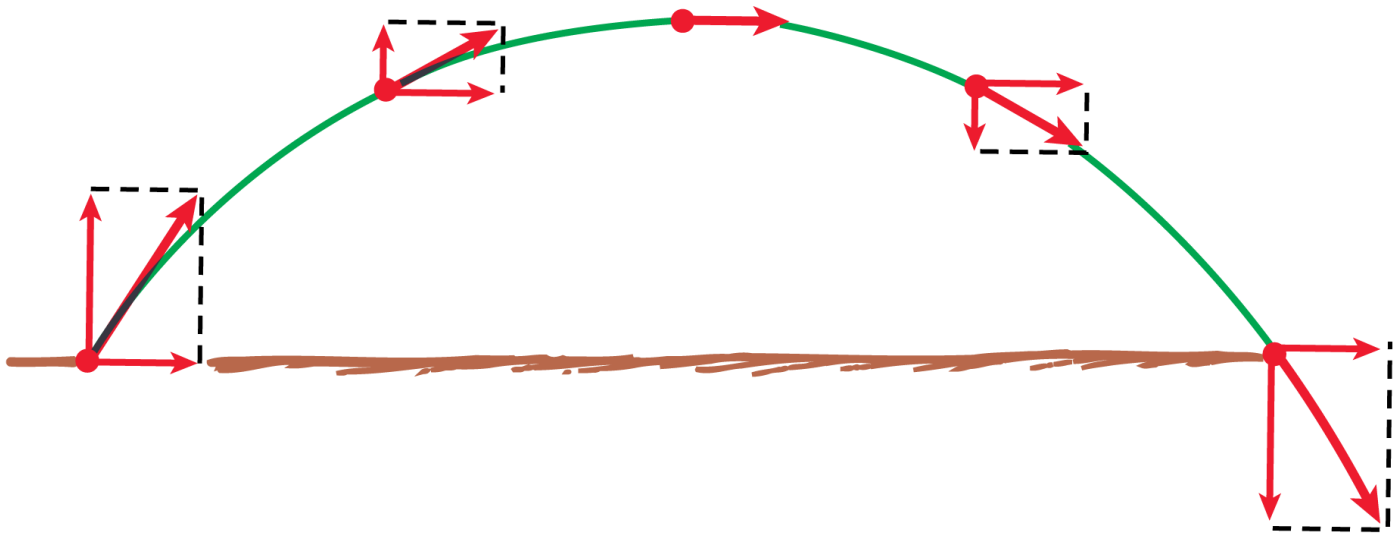
But because of gravity it falls beneath this line the same vertical distance it would fall if it were released from rest.



Projectiles Launched at an Angle

Velocity

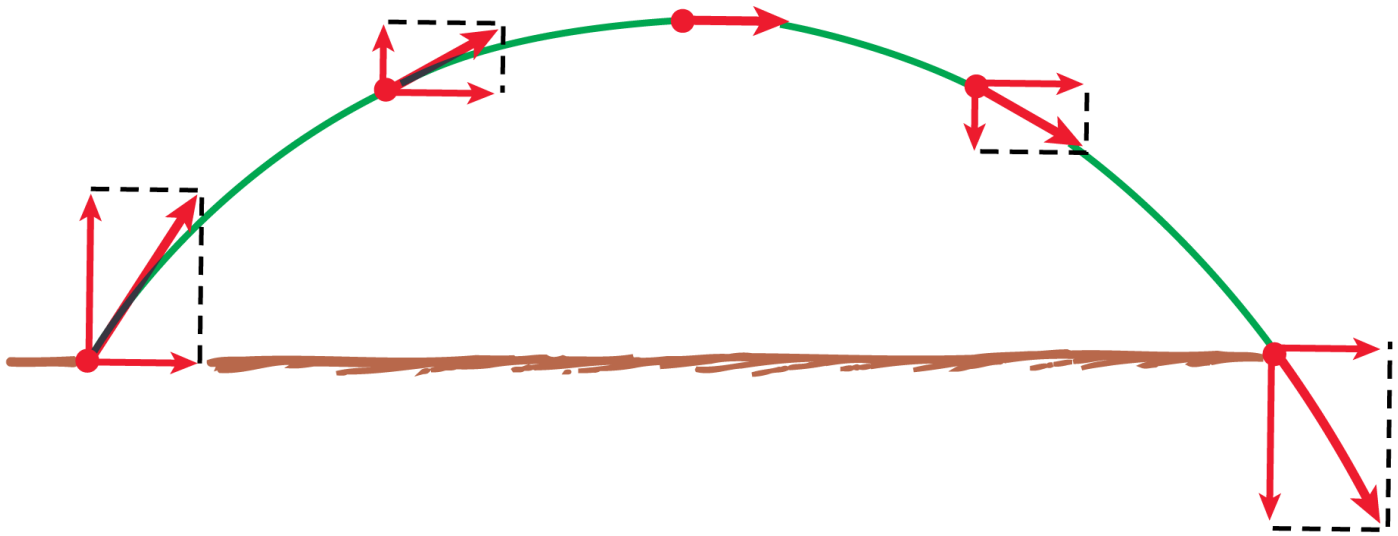
The velocity of a projectile is shown at various points along its path. Notice that the vertical component changes while the horizontal component does not. Air resistance is neglected.



Projectiles Launched at an Angle

Height

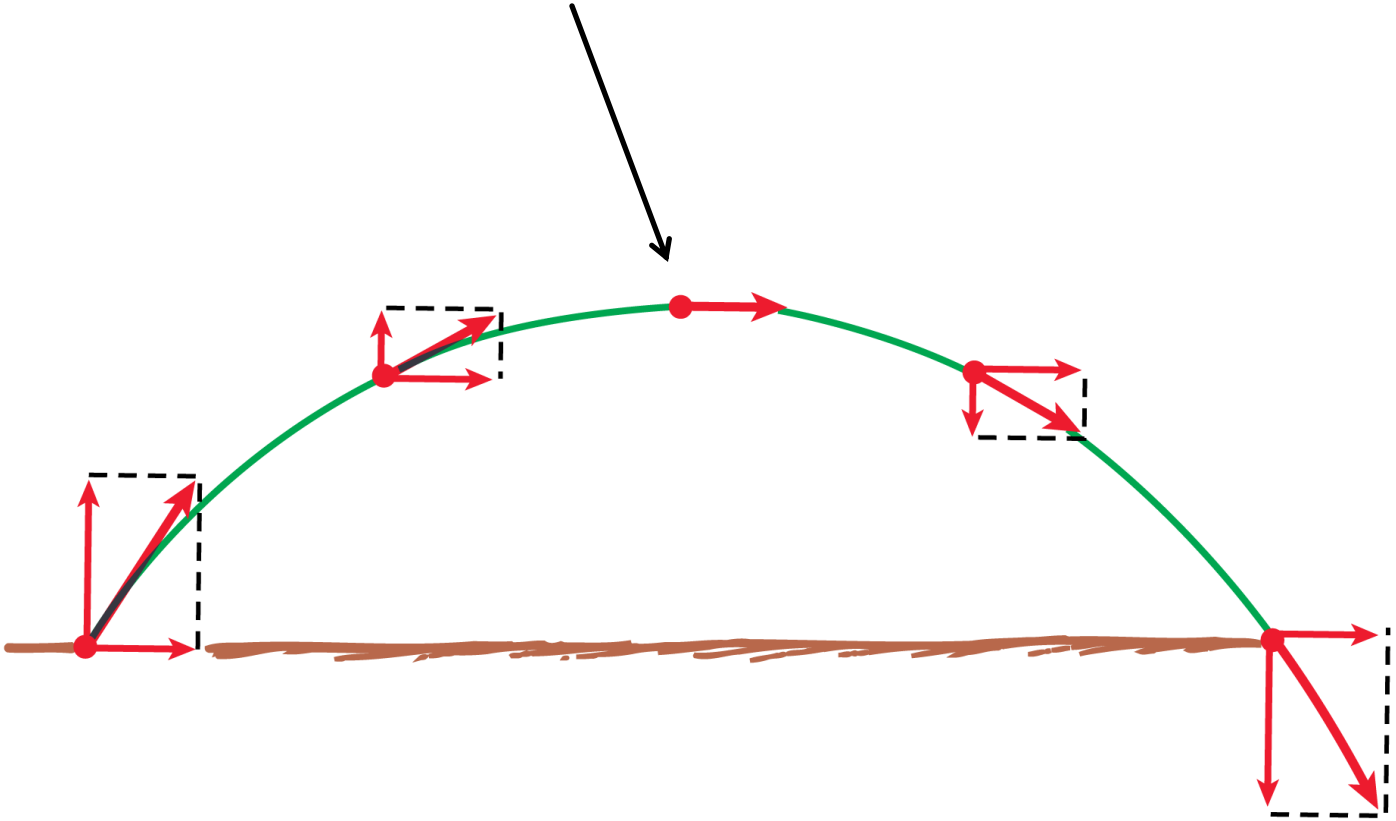
For the component vectors of the angled projectile motion, the horizontal component is always the same and only the vertical component changes.



Projectiles Launched at an Angle

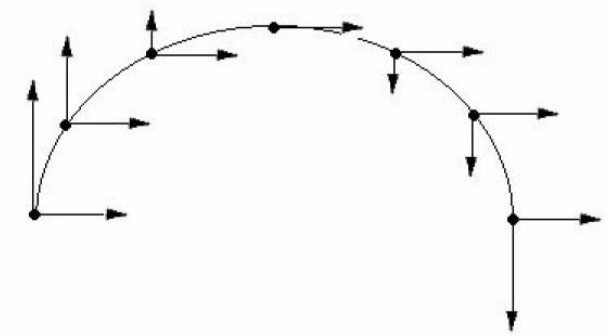
Height

At the top of the path the vertical component shrinks to zero.



Projectiles Launched at an Angle

Together, these components produce what is called a trajectory or path. This path is parabolic in nature.

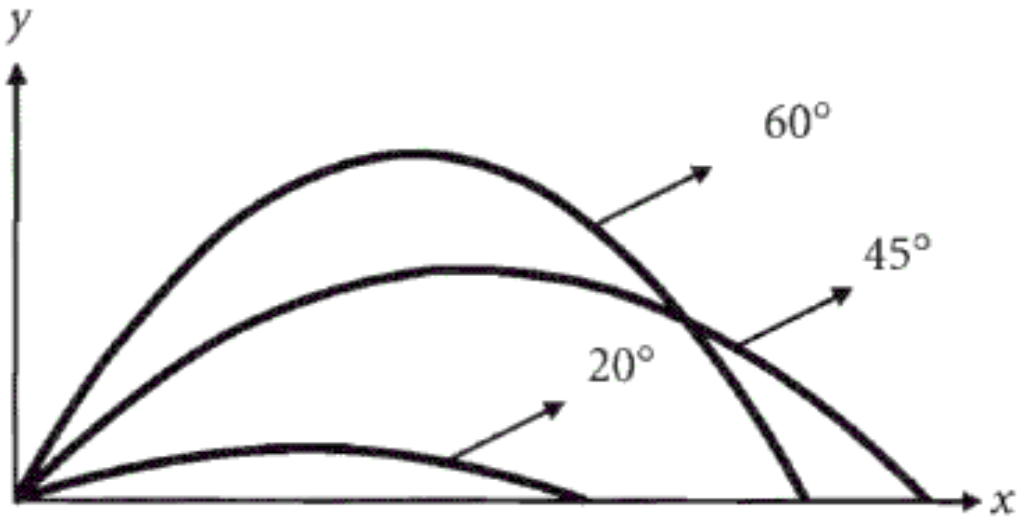


Component	Magnitude	Direction
Horizontal	Constant	Constant
Vertical	Changes	Changes

Projectiles Launched at an Angle

Range

The angle at which the projectile is launched affects the distance that it travels.



Projectiles Launched at an Angle

Horizontal Ranges

Projectiles that are launched at the same speed but at different angles reach different heights (altitude) above the ground.

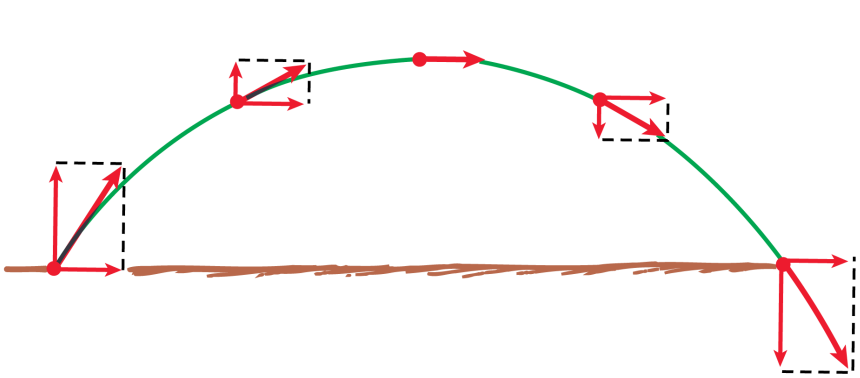
They also travel different horizontal distances, that is, they have different horizontal ranges.

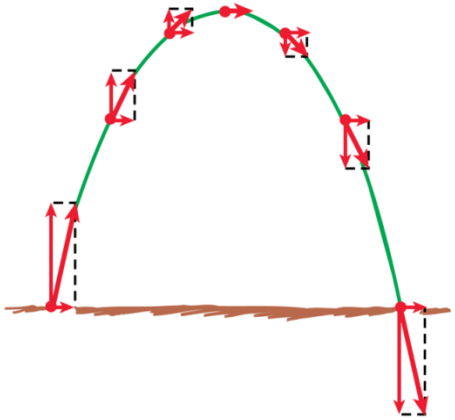
Projectiles Launched at an Angle

Horizontal Range

Both projectiles have the same launching speed.
The initial velocity vector has a greater vertical component than when the projection angle is less.

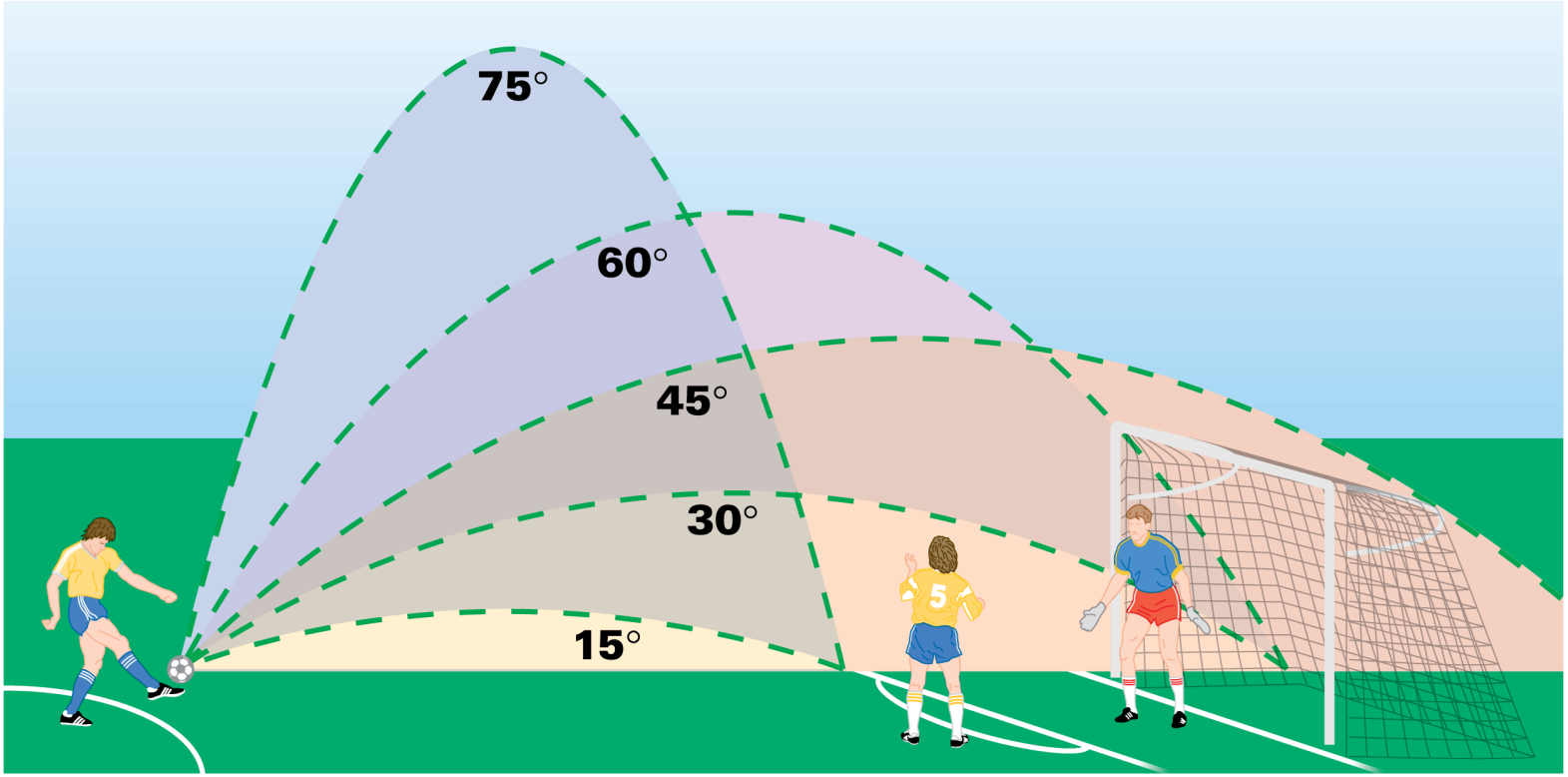
- **This greater component results in a higher path.**
- **The horizontal component is less, so the range is less.**





Projectiles Launched at an Angle

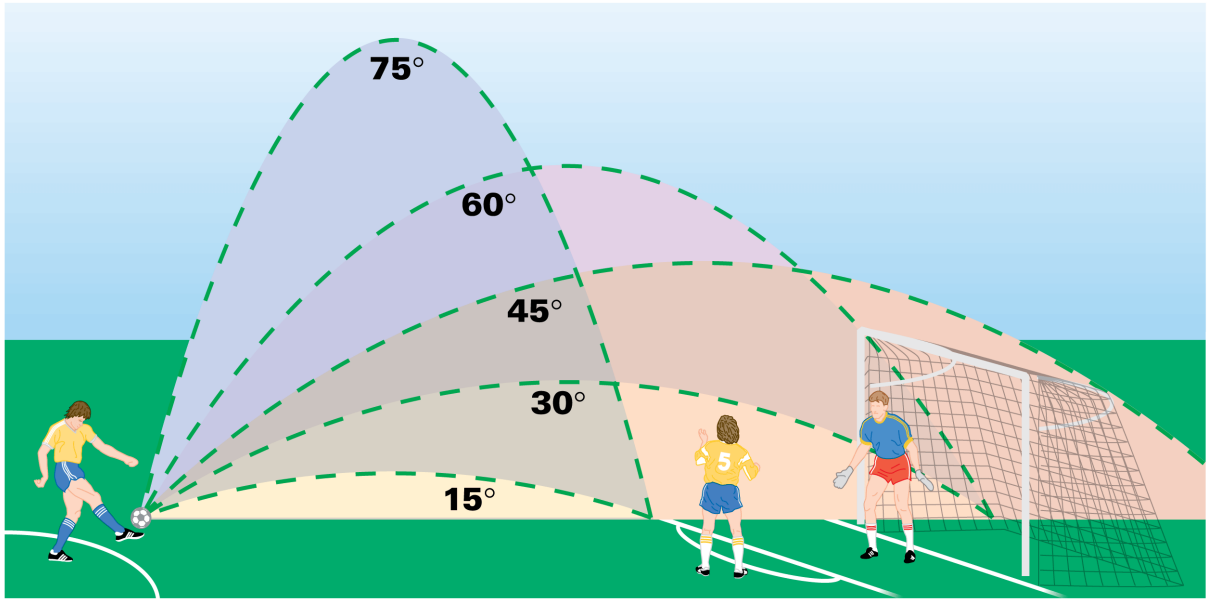
The paths of projectiles launched at the same speed but at different angles. The paths neglect air resistance.



Projectiles Launched at an Angle

The same range is obtained for two different projection angles—angles that add up to 90°.

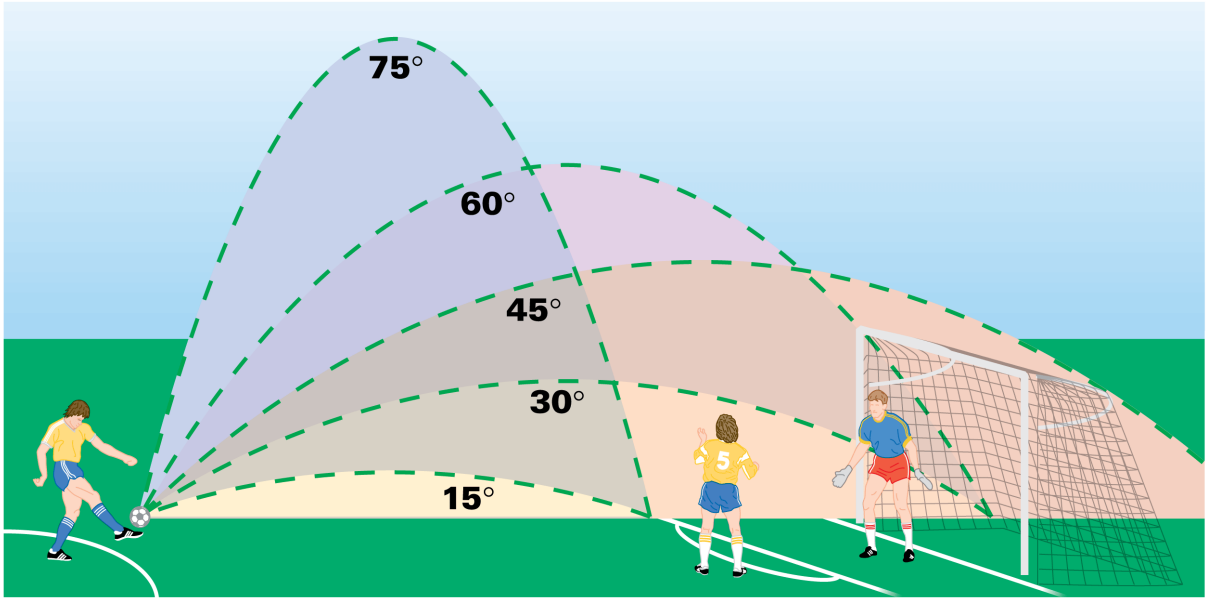
An object thrown into the air at an angle of 60° will have the same range as at 30° with the same speed.



Projectiles Launched at an Angle

Max Range

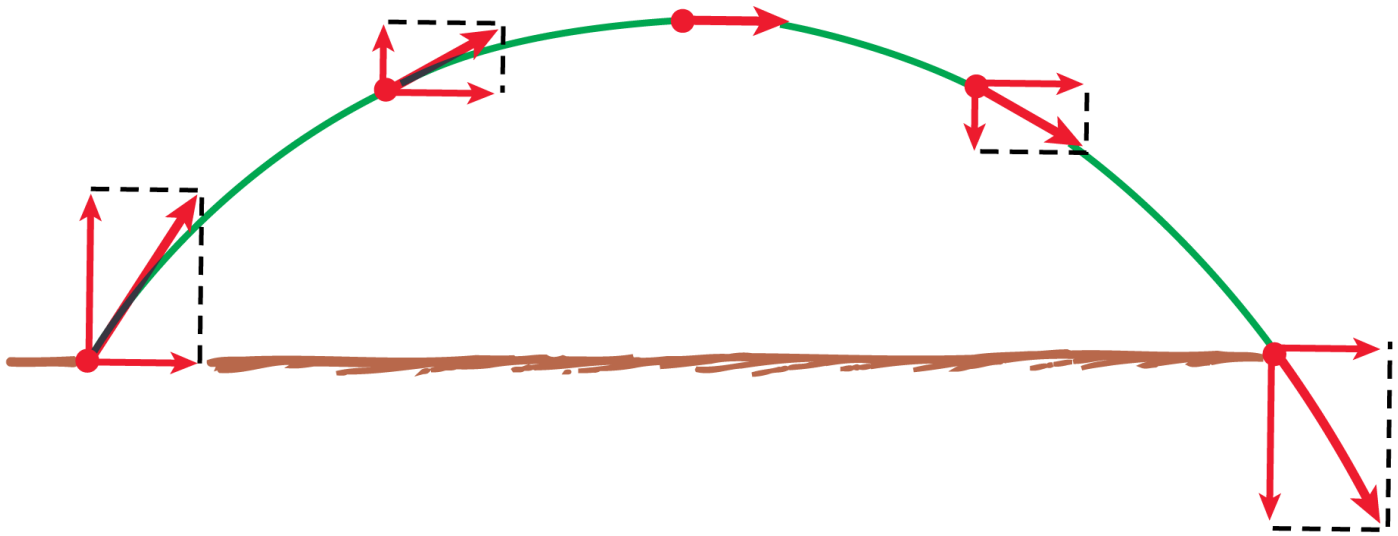
Maximum range is attained at an angle of 45°



Projectiles Launched at an Angle

Max Height

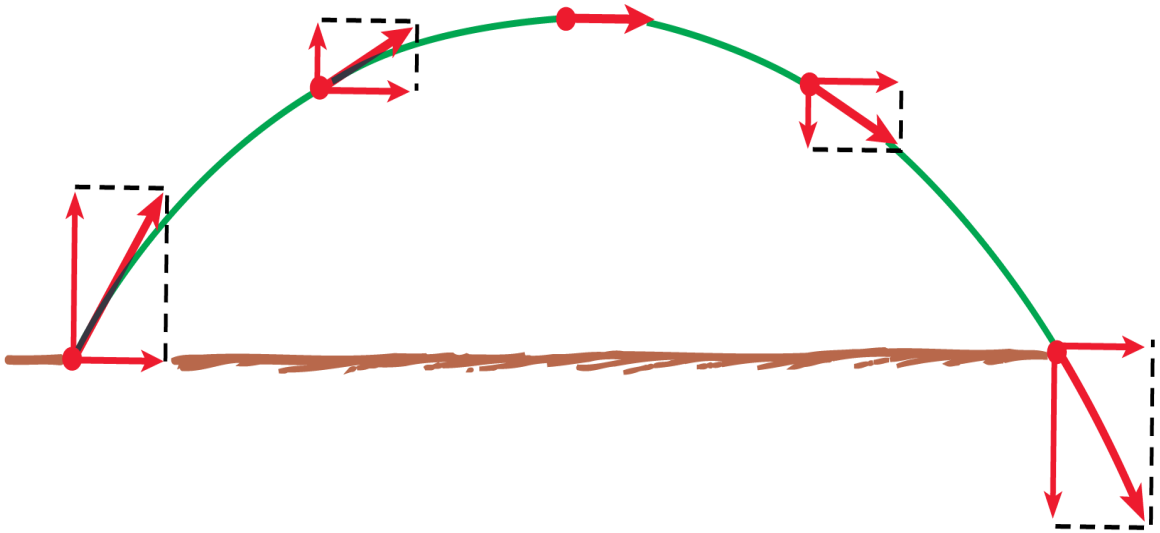
Without air resistance, a projectile will reach maximum height in the same time it takes to fall from that height to the ground.



Projectiles Launched at an Angle

Acceleration

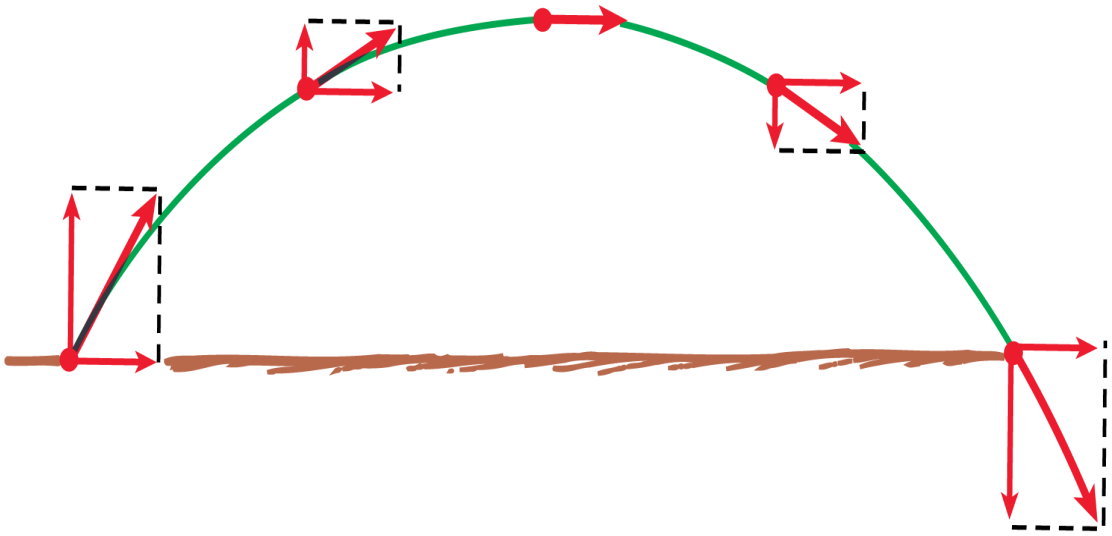
The deceleration due to gravity going up is the same as the acceleration due to gravity coming down.



Projectiles Launched at an Angle

Speed

The projectile hits the ground with the same speed it had when it was projected upward from the ground provided it lands with the a displacement of zero. (Lands at the same distance from the ground that it took off from)



Projectiles Launched at an Angle

Thinker!

A projectile is launched at an angle into the air. Neglecting air resistance, what is its vertical acceleration? Its horizontal acceleration?

Answer

Vertical acceleration = -9.81 m/s^2 or -32 ft/s^2

Horizontal acceleration = zero

Projectiles Launched at an Angle

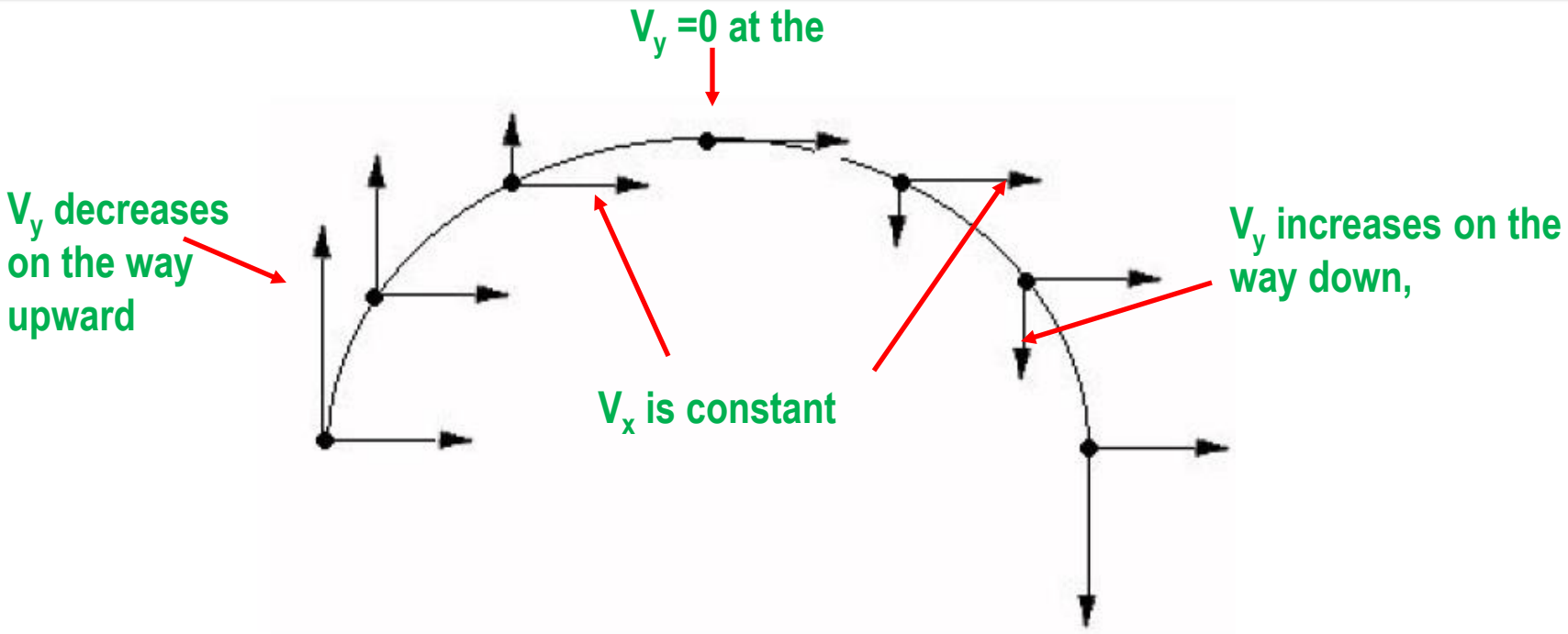
Thinker!

At what point in its path does a projectile have minimum speed?

Answer

Top of the parabolic path

Projectiles Launched at an Angle



Component	*Magnitude*	*Direction*
Horizontal	Constant	Constant
Vertical	• Decreases up • 0 @ top • Increases down	Changes

Projectiles Launched at an Angle

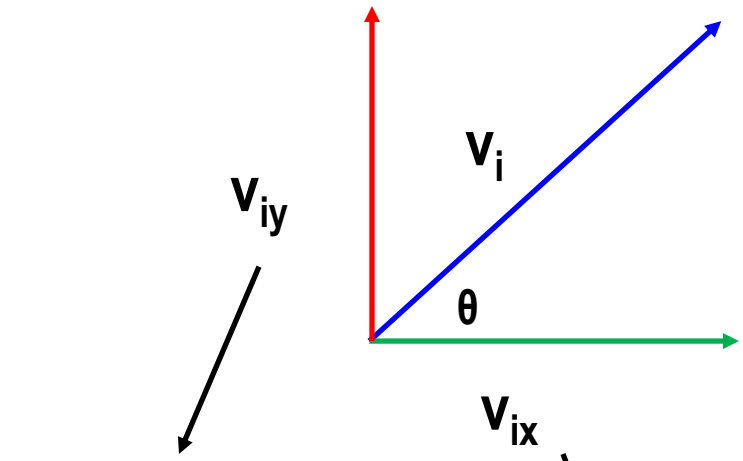
Projectiles launched at angles summary:

- The horizontal velocity is constant.
- It rises and falls in equal time intervals.
- It reaches maximum height in half the total time.
- Gravity only affects the vertical motion.
- If it begins and ends at ground level, the "d_y" displacement is ZERO: $d_y = 0$

Projectiles Launched at an Angle

Components

Since the projectile was launched at an angle, the velocity MUST be broken into components!!!



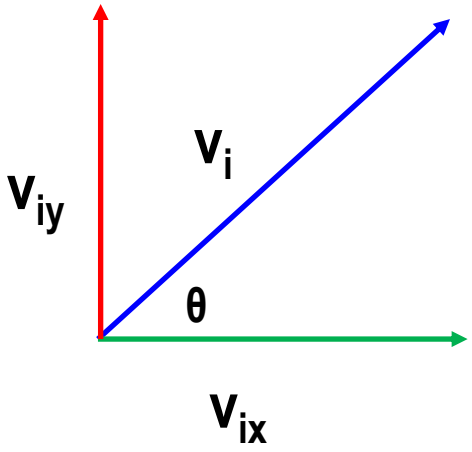
$V_{iy} = V_i \sin \theta$

$V_{ix} = V_i \cos \theta$

Projectiles Launched at an Angle

Formulas

You will still use the kinematic equations, but YOU MUST use COMPONENTS in the equation.



$d_x = V_{ix}t$	$d_y = V_{iy}t + \frac{1}{2}a_y t^2$

$V_{ix} = V_i \cos\theta$

$V_{iy} = V_i \sin\theta$

YOU WILL NEVER USE V_i IN YOUR KINEMATIC EQUATIONS!!!!!!!!!!!!

CONCEPT CHECK

What is the first thing you need to do if a projectile is launched at an angle?

You must break the initial velocity (V_i) into components:

V_{ix} and V_{iy}

Projectiles Launched at an Angle

Question #1

A place kicker kicks a football with a speed of 20 m/s and at an angle of 53°.

a. **What are the horizontal and vertical components of the initial speed?**
b. **How long is the ball in the air?**
c. **How far away does it land?**
d. **How high does it travel?**

Projectiles Launched at an Angle

Example #2

A body is projected upward from the level ground at an angle of 50° with the horizontal has an initial speed of 40 m/s.

a. What are the horizontal and vertical components of the initial speed?
b. How long will it be before it hits the ground?
c. How far from the starting point will the object hit the ground?
d. What is the maximum height it reached in the air?

a. $V_{ix} = 25.71$ m/s; $V_{iy} = 30.64$ m/s
b. 6.25 s
c. $d_x = 160.62$ m
d. $d_{ymax} = 47.85$ m